V_{us} from hyperon semileptonic decays

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Abstract

A model-independent determination of the CKM matrix element V_{us} from five measured strangeness-changing hyperon semileptonic decays is performed. Flavor $SU(3)$ symmetry breaking effects in the leading vector and axial-vector form factors are analyzed in the framework of the $1/N_c$ expansion of QCD. A fit to experimental data allows one to extract the value $V_{us} = 0.2199 \pm 0.0026$, which is comparable to the one from K_{e3} decays. This reconciliation is achieved through second-order symmetry breaking effects of a few percent in the form factors f_1, which increase their magnitudes over their $SU(3)$ predictions.

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I. INTRODUCTION

Hyperon semileptonic decays (HSD) play a decisive role in our understanding of the interplay between weak and strong interactions and the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix. At present, the determinations of V_{ud} and V_{us} provide the most precise constraints on the size of the CKM matrix elements. It has been argued that K_{e3} decays offer possibly the cleanest way to extract a precise value of V_{us} rather than HSD. From the theoretical point of view, the leptonic part of both semileptonic processes is unambiguous. In contrast, the hadronic part is deeply affected by flavor $SU(3)$ symmetry breaking in the form factors. For K_{e3} decays, this is a minor problem because only the vector part of the weak current has a nonvanishing contribution and only two form factors appear. In addition, such form factors are protected by the Ademollo-Gatto theorem [1] against $SU(3)$ breaking corrections to lowest order in $(m_s - \hat{m})$ so that the theoretical approach to compute them is under reasonable control within the limits of experimental precision. On the contrary, HSD are considerably more complicated than K_{e3} decays due to the participation of vector and axial-vector currents, which leads to the appearance of many more form factors. Although the leading vector form factors are also protected by the Ademollo-Gatto theorem, the analysis of HSD data has larger theoretical uncertainties because of first-order $SU(3)$ breaking effects in the axial-vector form factors.

Indeed, the current value of V_{us} recommended by the Particle Data Group [2] is the one from K_{e3} decays, namely,

$$V_{us} = 0.2200 \pm 0.0026. \tag{1}$$

Recent studies of K_{e3} decays [3, 4, 5], HSD [6], and lattice gauge theory [7] suggest larger values of V_{us}, in disagreement with early determinations [8]. This discrepancy is an outstanding problem and should be addressed.

Inspired by those facts, in this paper we perform a detailed model-independent analysis of the determination of V_{us} from five already observed $|\Delta S| = 1$ HSD. The goals in performing this study are to confirm the value of V_{us} obtained from K_{e3} decays, Eq. (1), and to use the form factors to achieve a better understanding of the hadronic structure.

In order to have a precise and reliably determination of V_{us}, we systematically consider two major approaches. First, we incorporate radiative corrections to various measurable quantities relevant for experimental analyses and include the momentum-transfer contributions of the form factors. And second, we analyze $SU(3)$ symmetry breaking effects into the leading vector and

axial-vector form factors in the framework of the $1/N_c$ expansion of QCD, following the lines of Ref. [9]. The resultant theoretical expressions are thus compared with the available experimental data on HSD [2], allowing an extraction of V_{us}. Here we need to point out a slight difference between our procedure and the one of Ref. [9]. There, a global fit of HSD and pionic decays of the decuplet baryons was performed, whereas in our case we concentrate only on the $|\Delta S| = 1$ sector.

This work is organized as follows. In Sec. II we provide some theoretical issues on HSD. In Sec. III we give a general overview of the $1/N_c$ expansion of baryon operators whose matrix elements yield the HSD form factors. In Sec. IV-VI we perform detailed comparisons of the theoretical expressions with the current experimental data on HSD [2] through several fits under various assumptions. We present results and conclusions in Sec. VII. In Appendix A we provide numerical formulas for the integrated observables used in our analysis.

II. HYPERON SEMILEPTONIC DECAYS

In this section we will review our notation and conventions. For definiteness, let us consider the hyperon semileptonic decay

$$B_1 \to B_2 + \ell + \overline{\nu}_l, \tag{2}$$

where B_1 and B_2 are spin-1/2 hyperons, ℓ is the charged lepton ($\ell = e, \nu$), and $\overline{\nu}_\ell$ is the accompanying antineutrino or neutrino, as the case may be. The four-momenta and masses of the particles involved in process (2) are denoted hereafter by $p_1 = (E_1, \mathbf{p}_1)$ and M_1, $p_2 = (E_2, \mathbf{p}_2)$ and M_2, $l = (E, \mathbf{l})$ and m, and $p_\nu = (E_\nu^0, \mathbf{p}_\nu)$ and m_ν, respectively.

The low-energy weak interaction Hamiltonian for semileptonic processes reads

$$H_W = \frac{G}{\sqrt{2}} J_\alpha L^\alpha + \mathrm{H.c.}, \tag{3}$$

where L_α and J_α denote the leptonic and hadronic currents, respectively. The former is given by

$$L^\alpha = \overline{\psi}_e \gamma^\alpha (1-\gamma_5)\psi_{\nu_e} + \overline{\psi}_\mu \gamma^\alpha (1-\gamma_5)\psi_{\nu_\mu}, \tag{4}$$

whereas J_α, expressed in terms of the vector (V_α) and axial-vector (A_α) currents, can be written as

$$J_\alpha = V_\alpha - A_\alpha, \tag{5a}$$

$$V_\alpha = V_{ud}\overline{u}\gamma_\alpha d + V_{us}\overline{u}\gamma_\alpha s, \tag{5b}$$

$$A_\alpha = V_{ud}\overline{u}\gamma_\alpha\gamma_5 d + V_{us}\overline{u}\gamma_\alpha\gamma_5 s. \tag{5c}$$

Here G is the weak coupling constant, and V_{ud} and V_{us} are the appropriate elements of the CKM matrix.

The matrix elements of J_α between spin-$1/2$ states can be written as

$$\langle B_2|V_\alpha|B_1\rangle \;=\; V_{\rm CKM}\,\overline{u}_{B_2}(p_2)\left[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{M_1}\sigma_{\alpha\beta}q^\beta + \frac{f_3(q^2)}{M_1}q_\alpha\right]u_{B_1}(p_1), \tag{6}$$

$$\langle B_2|A_\alpha|B_1\rangle \;=\; V_{\rm CKM}\,\overline{u}_{B_2}(p_2)\left[g_1(q^2)\gamma_\alpha + \frac{g_2(q^2)}{M_1}\sigma_{\alpha\beta}q^\beta + \frac{g_3(q^2)}{M_1}q_\alpha\right]\gamma_5 u_{B_1}(p_1), \tag{7}$$

where $q \equiv p_1 - p_2$ is the four-momentum transfer, u_{B_1} and $\overline{u}_{B_2}$ are the Dirac spinors of the corresponding hyperons, and $V_{\rm CKM}$ is either V_{ud} or V_{us}. In this work we adopt the metric and γ-matrix conventions of Ref. [10]. The quantities $f_1(q^2)$ and $g_1(q^2)$ are the vector and axial-vector form factors, $f_2(q^2)$ and $g_2(q^2)$ are the weak magnetism and electricity form factors, and $f_3(q^2)$ and $g_3(q^2)$ are the induced scalar and pseudoscalar form factors, respectively. Time reversal invariance requires the form factors to be real. $f_3(q^2)$ and $g_3(q^2)$, for electron or positron emission, have negligible contributions to the decay rate due to the smallness of the factor $(m/M_1)^2$ which comes along with them. Therefore, to a high degree of accuracy, the e-modes of HSD are described in terms of four, rather than six, form factors. In contrast, for μ-modes although the factor $(m/M_1)^2$ is still small, $f_3(q^2)$ and $g_3(q^2)$ may contribute with some significance and should be retained. For convenience, here we introduce the definitions $f_i \equiv f_i(0)$ and $g_i \equiv g_i(0)$, with $i = 1, 2, 3$.

A. Differential decay rate

The transition amplitude for process (2) can be constructed from the product of the matrix elements of the hadronic and leptonic currents [10]. From this amplitude, the differential decay rate of HSD, denoted here by $d\Gamma$, can be derived by using standard techniques [10, 11]. For the three-body decay (2) different choices of the five relevant variables in the final states will lead to appropriate expressions for $d\Gamma$. In Ref. [10], for instance, detailed expressions have been obtained for $d\Gamma$ in the rest frame of B_1 $[B_2]$ when such hyperon is polarized along the direction s_1 $[s_2]$, and with the charged lepton ℓ and neutrino going into the solid angles $d\Omega_\ell$ and $d\Omega_\nu$, respectively. Similarly, in Refs. [11, 12] $d\Gamma$ has been obtained, in the rest frame of B_1, by leaving the electron and emitted hyperon energies as the relevant variables along with some suitable angular variables.

In all the above cases the differential decay rate can be written, in the most general case, as

$$d\Gamma = G^2 d\Phi_3\left[A_0' - A_0''\,\hat{\mathbf{s}}\cdot\hat{\mathbf{p}}\right], \tag{8}$$

where $d\Phi_3$ is an element of the appropriate three-body phase space and A_0' and A_0'' depend on the kinematical variables and are quadratic functions of the form factors. The scalar product $\hat{\mathbf{s}} \cdot \hat{\mathbf{p}}$, where $\hat{\mathbf{s}}$ denotes the spin of either B_1 or B_2 and $\hat{\mathbf{p}} = \hat{\mathbf{l}}, \hat{\mathbf{p}}_2, \hat{\mathbf{p}}_\nu$, represents the angular correlation between such spin and the three-momentum of the corresponding particle [11, 12].

B. Integrated observables

When experiments in HSD have low statistics one cannot perform a detailed analysis of the differential decay rate $d\Gamma$. One is thus led to produce some integrated observables instead, namely, the total decay rate R and angular correlation and asymmetry coefficients. The definitions of these observables entail only kinematics and do not assume any particular theoretical approach. For example, the charged lepton-neutrino angular correlation coefficient is defined as

$$\alpha_{\ell\nu} = 2\frac{N(\Theta_{\ell\nu} < \pi/2) - N(\Theta_{\ell\nu} > \pi/2)}{N(\Theta_{\ell\nu} < \pi/2) + N(\Theta_{\ell\nu} > \pi/2)}, \tag{9}$$

where $N(\Theta_{\ell\nu} < \pi/2)$ $[N(\Theta_{\ell\nu} > \pi/2)]$ is the number of charged lepton-neutrino pairs emitted in directions that make an angle between them smaller [greater] than $\pi/2$. Similar expressions can be derived for the charged lepton α_ℓ, neutrino α_ν, and emitted hyperon α_B asymmetry coefficients, this time Θ_ℓ, Θ_ν, and Θ_B being the angles between the ℓ, ν, and B_2 directions and the polarization of B_1, respectively. When the polarization of the emitted hyperon is observed, two more asymmetry coefficients, A and B, can be defined [10]. If the charged lepton mass can be neglected it is rather straightforward to compute approximate theoretical expressions for these observables. All this has been done in Ref. [10] for a number of decays. For the uncorrected total decay rate one has

$$\begin{aligned} R^0 \;=\; & G^2\frac{(\Delta M)^5}{60\pi^3}\left[\left(1-\frac{3}{2}\beta+\frac{6}{7}\beta^2\right)f_1^2+\frac{4}{7}\beta^2 f_2^2+\left(3-\frac{9}{2}\beta+\frac{12}{7}\beta^2\right)g_1^2\right. \\ & \left. +\,\frac{12}{7}\beta^2 g_2^2+\frac{6}{7}\beta^2 f_1 f_2+(-4\beta+6\beta^2)g_1 g_2\right], \end{aligned} \tag{10}$$

where $\beta = (M_1 - M_2)/M_1$ and the superscript 0 on a given observable is used as an indicator that no radiative corrections have been incorporated into it. In Eq. (10), although the form factors have been assumed to be constant, their q^2-dependence cannot always be neglected since they can give a noticeable contribution. In order to obtain expressions correct to order $\mathcal{O}(q^2)$, the q^2-dependence of f_2 and g_2 can be ignored because they already contribute to order $\mathcal{O}(q)$ to the decay rate. For

$f_1(q^2)$ and $g_1(q^2)$, however, a linear expansion in q^2 is enough because higher powers amount to negligible contributions to the decay rate, no larger than a fraction of a percent. Thus,

$$f_1(q^2) = f_1(0) + \frac{q^2}{M_1^2}\lambda_1^f, \qquad\qquad g_1(q^2) = g_1(0) + \frac{q^2}{M_1^2}\lambda_1^g, \tag{11}$$

where the slope parameters λ_1^f and λ_1^g are both of order unity [10]. A dipole parametrization for the leading form factors such as $f(q^2) = f(0)/(1 - q^2/M^2)^2$ yields

$$\lambda_1^f = \frac{2M_1^2 f_1}{M_V^2}, \qquad\qquad \lambda_1^g = \frac{2M_1^2 g_1}{M_A^2}, \tag{12}$$

where $M_V = 0.97$ GeV and $M_A = 1.11$ GeV for $|\Delta S| = 1$ HSD [10].

For more precise formulas and when the charged lepton mass is retained, one needs to numerically integrate over the kinematical variables the expressions for $d\Gamma$ and angular coefficients already given in previous works [10, 11, 12]. Concerning this, Ref. [10] provides complete numerical formulas for the decay rates and angular coefficients of the 16 e-mode and 10 μ-mode HSD. These formulas, however, are almost 20 years old and the current experimental data on hyperon masses [2] introduce modifications to them which need to be accounted for. We have recalculated and updated the formulas for the uncorrected integrated observables of five HSD we are concerned with in the present analysis. They are listed in Appendix A for the sake of completeness.

C. Radiative corrections

Experiments on HSD have gradually become sensitive enough to require radiative corrections to the integrated observables. However, the calculation of radiative corrections to processes involving hadrons has been a long standing problem. Despite the outstanding progress achieved in the understanding of the fundamental interactions with the Standard Model [2], no first principle calculation of radiative corrections is yet possible. These corrections thus become committed to model dependence and experimental analyses that use them also become model dependent. Even if the model dependence arising from the virtual radiative corrections cannot be eliminated, an analysis in neutron beta decay further extended to HSD [13] shows that to orders $(\alpha/\pi)(q/M_1)^0$ and $(\alpha/\pi)(q/M_1)$ such model-dependence amounts to some constants, which can be absorbed into the form factors originally defined in the matrix elements of the hadronic current. In addition, the theorem of Low in its version by Chew [14] can be used to show that to these two

orders of approximation the bremsstrahlung radiative corrections depend only on both the non-radiative form factors and the static electromagnetic multipoles of the particles involved so that no model-dependence appears in this other part of the radiative corrections. Within these orders of approximation one is left with general expressions which can be used in model-independent analyses [10, 11, 12].

The radiative corrections to order $(\alpha/\pi)(q/M_1)^0$ to all the integrated observables of HSD referred to above have been computed in Ref. [10]. There it was shown that to this order of approximation, the angular and asymmetry coefficients for both e- and μ-mode do not get affected by these corrections, so to a good approximation, $\alpha \simeq \alpha^0$, where α stands for any of the angular coefficients considered here. In contrast, the total decay rate R is corrected as $R = R^0[1+(\alpha/\pi)\Phi]$, where R^0 is the uncorrected decay rate and Φ comes from the model-independent part of radiative corrections. The function Φ can be obtained from Eqs. (5.25) and (5.28) of Ref. [10]; their numerical values for several decays are listed in Table 5.1 of that reference. We have also numerically evaluated Φ for several HSD and found a very good agreement with the values already obtained so we will not repeat them here.

As for the model dependent part of radiative corrections, we cannot compute it rigorously. Reference [10], however, proposes as a parametrization of this model dependence a modified weak coupling constant $G \equiv G(1 + C)$, where $C \sim 0.0234$. This value of C could give a noticeable contribution to the total decay rate. We will adopt this approach in the present analysis.

D. Experimental data on HSD

The experimentally measured quantities [2] in HSD are the total decay rate R, angular correlation coefficients $\alpha_{e\nu}$, and angular spin-asymmetry coefficients α_e, α_ν, α_B, A, and B. An alternative set of experimental data is constituted by the decay rates and measured g_1/f_1 ratios. This latter set, however, is not as rich as the former and will not be used in the present analysis, unless noted otherwise. Currently there are five HSD which have sufficient data to reliably extract the value of V_{us}. These processes are $\Lambda \to pe^-\overline{\nu}_e$, $\Sigma^- \to ne^-\overline{\nu}_e$, $\Xi^- \to \Lambda e^-\overline{\nu}_e$, $\Xi^- \to \Sigma^0 e^-\overline{\nu}_e$, and $\Xi^0 \to \Sigma^+ e^-\overline{\nu}_e$. Their available experimental information is displayed in Table I.

TABLE I: Experimental data on five measured $|\Delta S| = 1$ HSD. The units of R are 10^6 s^{-1}.

	$\Lambda \to pe^-\overline{\nu}_e$	$\Sigma^- \to ne^-\overline{\nu}_e$	$\Xi^- \to \Lambda e^-\overline{\nu}_e$	$\Xi^- \to \Sigma^0 e^-\overline{\nu}_e$	$\Xi^0 \to \Sigma^+ e^-\overline{\nu}_e$
R	3.161 ± 0.058	6.88 ± 0.24	3.44 ± 0.19	0.53 ± 0.10	0.93 ± 0.14
$\alpha_{e\nu}$	-0.019 ± 0.013	0.347 ± 0.024	0.53 ± 0.10		
α_e	0.125 ± 0.066	-0.519 ± 0.104			
α_ν	0.821 ± 0.060	-0.230 ± 0.061			
α_B	-0.508 ± 0.065	0.509 ± 0.102			
A			0.62 ± 0.10		
g_1/f_1	0.718 ± 0.015	-0.340 ± 0.017	0.25 ± 0.05	1.287 ± 0.158	1.32 ± 0.22

III. HSD FORM FACTORS IN THE $1/N_c$ EXPANSION OF QCD

In the past when data were not very precise, fits to HSD were made under the assumption of exact $SU(3)$ symmetry in order to extract V_{us}. Currently, the experiments are precise enough to the extent that this assumption no longer provides a reliably fit. Therefore, the determination of V_{us} from HSD requires an understanding of the $SU(3)$ symmetry breaking effects in the weak form factors. We devote this section to evaluate these effects within the framework of the $1/N_c$ expansion of QCD. The form factors are analyzed in a combined expansion in $1/N_c$ and $SU(3)$ symmetry breaking following the lines of Refs. [9, 15, 16]. Before doing so we first review some necessary large-N_c formalism.

For large N_c, the lowest-lying baryons are given by the completely symmetric spin-flavor representation of N_c quarks. Under $SU(2) \times SU(N_F)$, this $SU(2N_F)$ representation decomposes into a tower of baryon flavor representations with spins $J = \frac{1}{2}, \frac{3}{2}, \ldots, \frac{N_c}{2}$. For two flavors of light quarks the baryon tower consists of (spin,isospin) representations with $I = J$, whereas for three flavors the baryon flavor representations become much more complex [15, 17].

In order to simplify the analysis, it is much better to concentrate on the baryon operators, rather than on the states, because the former have a simple expansion in $1/N_c$ for arbitrary N_c. In this context, the general form of the $1/N_c$ expansion of a QCD m-body quark operator acting on a single baryon state can be written as [15, 17]

$$\mathcal{O}_{\text{QCD}}^{m\text{-body}} = N_c^m \sum_{n=0}^{N_c} c_n \frac{1}{N_c^n} \mathcal{O}_n, \tag{13}$$

where c_n are unknown coefficients which have power series expansions in $1/N_c$ beginning at order unity. The sum in Eq. (13) is over all possible independent n-body operators $\mathcal{O}_n$, $0 \leq n \leq N_c$, with

the same spin and flavor quantum numbers as $\mathcal{O}_{\rm QCD}$. The use of operator identities [15] reduces the operator basis to independent operators. The large-N_c spin-flavor symmetry for baryons is generated by the baryon spin, flavor and spin-flavor operators J^i, T^a, and G^{ia} which can be written for large but finite N_c as one-body quark operators acting on the N_c-quark baryon states as

$$J^i = q^\dagger \left(\frac{\sigma^i}{2} \otimes \mathbb{1}\right) q \qquad (1,1), \tag{14a}$$

$$T^a = q^\dagger \left(\mathbb{1} \otimes \frac{\lambda^a}{2}\right) q \qquad (0,8), \tag{14b}$$

$$G^{ia} = q^\dagger \left(\frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2}\right) q \qquad (1,8). \tag{14c}$$

The transformation properties of these generators under $SU(2) \times SU(3)$ are given explicitly in Eq. (14) as (j,d), where j is the spin and d is the dimension of the $SU(3)$ flavor representation.

In this paper we analyze the $1/N_c$ expansions of the QCD baryon vector and axial vector currents whose matrix elements between $SU(6)$ symmetric states give the HSD form factors. The detailed analysis has already been done [9, 16], so we will limit ourselves to only state the answer here.

A. Vector form factor f_1

At $q^2 = 0$ the hyperon matrix elements for the vector current are given by the matrix elements of the associated charge or $SU(3)$ generator. The flavor octet baryon charge is denoted by [9]

$$V^{0a} = \left\langle B_2 \left| \left(\overline{q}\gamma^0 \frac{\lambda^a}{2} q\right)_{\rm QCD} \right| B_1 \right\rangle \tag{15}$$

and its matrix elements between $SU(6)$ symmetric states yield the value of f_1. V^{0a} is spin-0 and a flavor octet so that it transforms as (0,8) under $SU(2) \times SU(3)$. The $1/N_c$ expansion for the baryon vector current in the limit of exact $SU(3)$ symmetry has the form

$$V^{0a} = \sum_{n=1}^{N_c} a_n \frac{1}{N_c^{n-1}} \mathcal{O}_n^a, \tag{16}$$

where the allowed one- and two-body operators are $\mathcal{O}_1^a = T^a$ and $\mathcal{O}_2^a = \{J^i, G^{ia}\}$. Higher order operators are obtained from the former as $\mathcal{O}_{n+2}^a = \{J^2, \mathcal{O}_n^a\}$. The fact that at $q^2 = 0$ the baryon vector current V^{0a} is the generator of $SU(3)$ symmetry transformations imposes $a_1 = 1$ and $a_n = 0$ for $n \geq 2$ in expansion (16). Therefore, in this limit one has [9]

$$V^{0a} = T^a, \tag{17}$$

whose matrix elements are denoted hereafter as $f_1^{SU(3)}$.

Flavor $SU(3)$ symmetry breaking in QCD is due to the light quark masses and transforms as a flavor octet. The $SU(3)$ symmetry breaking correction to V^{0a} was computed to second order in symmetry breaking in Ref. [9], as stated by the Ademollo-Gatto theorem [1]. The final expression for the $1/N_c$ expansion of V^{0a} can be cast into

$$V^{0a} = (1+v_1)T^a + v_2\{T^a, N_s\} + v_3\{T^a, -I^2 + J_s^2\}, \tag{18}$$

where v_i are parameters to be determined. Besides, N_s is the number of strange quarks, I is the isospin, and J_s is the strange quark spin. The matrix elements of the operators involved in the expansion (18) can be found in Ref. [9] as well.

B. Axial vector form factor g_1

The $1/N_c$ expansion for the baryon axial-vector current A^{ia} was first discussed in Refs. [15, 16]. We will use a simplified version of their results here. For the $|\Delta S| = 1$ sector of HSD, A^{ia} can be written as

$$\frac{1}{2}A^{ia} = a'G^{ia} + b'J^iT^a + c_3\{G^{ia}, N_s\} + c_4\{T^a, J_s^i\}. \tag{19}$$

Previous works [9, 16] included an extra term in expansion (19) to account for strangeness-zero decays. Adding this term avoided the mixing between symmetry breaking effects and $1/N_c$ corrections in the symmetric couplings D, F, and $\mathcal{C}$. In our case such a term in not necessary, so we have removed it and kept only those terms which contribute to strangeness-changing processes. This results in redefinitions of the parameters a and b of these references into a' and b', which absorb the terms c_1 and c_2, respectively, of the original expansion. The couplings D and F have to be redefined accordingly. For A^{ia} we are thus left with four parameters, namely, a', b', c_3 and c_4.

C. The form factors f_2 and g_2

The contributions of f_2 and g_2 to the decay amplitudes are suppressed by the momentum transfer. In the symmetry limit the hyperon masses are degenerate and then such contributions vanish. Thus, the first-order symmetry breaking corrections to f_2 and g_2 actually contribute to second order in the decay amplitude.

In the limit of exact $SU(3)$ flavor symmetry the form factor f_2 is described by two invariants, m_1 and m_2, which can be determined from the anomalous magnetic moments of the nucleons [16]. The magnetic moment is a spin-1 octet operator so it has a $1/N_c$ expansion identical in structure to the baryon axial-vector current A^{ia} [15, 16]. Nevertheless, it has been shown that reasonable shifts from the $SU(3)$ predictions of f_2 have no perceptible effects upon χ^2 or g_1 in a global fit to experimental data [18, 19]. We therefore follow these references and determine f_2 with the best fit values $m_1 = 2.87$ and $m_2 = -0.77$ [16].

As for the form factor g_2, it vanishes in the $SU(3)$ flavor symmetry limit, so it is proportional to $SU(3)$ symmetry breaking at leading order. The $1/N_c$ expansion for this form factor is given in detail in Ref. [9], where an attempt was made in order to extract some quantitative information about it. However, it was concluded that the experimental data are not precise enough for the extraction of the small g_2-dependence of the decay amplitudes. We take the value $g_2 = 0$ in our analysis accordingly.

IV. FITS TO EXPERIMENTAL DATA: DECAY RATES AND ANGULAR COEFFICIENTS

At this point we are now in a position to perform detailed comparisons with the experimental data of Table I through a number of fits. The experimental data which are used are the decay rates and the spin and angular correlation coefficients of the five HSD listed. The value of the ratio g_1/f_1 is not used since it is determined from other quantities and is not an independent measurement. For the processes $\Xi^- \to \Sigma^0 e^- \overline{\nu}_e$ and $\Xi^0 \to \Sigma^+ e^- \overline{\nu}_e$, however, we have no other choice but to use g_1/f_1 because no information on the angular coefficients is available yet. The theoretical expressions for the total decay rates and angular coefficients are organized in several tables in Appendix A. In the analysis we also take into account both model-independent and model-dependent radiative corrections and the q^2-dependence of the leading form factors, as stated in Sec. II.

The parameters to be fitted are those arising out of the $1/N_c$ expansions of the baryon operators whose matrix elements between $SU(6)$ symmetric states give the values of the couplings, namely, v_{1-3} for f_1 [introduced in Eq. (18)] and a', b', c_{3-4} for g_1 [introduced in Eq. (19)]. We use the values of f_2 and g_2 in the limit of exact $SU(3)$ flavor symmetry. An additional input is the value of V_{us}, Eq. (1), which is mainly the one from K_{e3} decays. We also extract information on V_{us} by fitting it as well. Hereafter, the quoted errors of the best fit parameters will be from the χ^2 fit only,

and will not include any theoretical uncertainties.

A. Exact $SU(3)$ symmetry

As a starting point we can perform a rough $SU(3)$ symmetric fit which involves only the parameters a' and b' for g_1. Our aim is not quite to test the $1/N_c$ predictions but rather to explore the quality of the data of Table I. The results are displayed in the second column of Table II, labeled as Fit 1(a). We can immediately notice some interesting results. As expected, the leading parameter a' is order unity and b' is order $1/N_c$, in good agreement with previous works [9, 16]. In this case $\chi^2 = 38.63$ for 15 degrees of freedom. From the χ^2 point of view, the fit is very poor. The large value of χ^2 is built up mainly by α_e ($\Delta\chi^2 = 2.83$) and α_ν ($\Delta\chi^2 = 6.89$) in $\Lambda \to pe^-\overline{\nu}_e$, R ($\Delta\chi^2 = 3.73$), α_ν ($\Delta\chi^2 = 4.04$), and α_B ($\Delta\chi^2 = 2.32$) in $\Sigma^- \to ne^-\overline{\nu}_e$, and finally R ($\Delta\chi^2 = 11.46$) and A ($\Delta\chi^2 = 2.05$) in $\Xi^- \to \Lambda e^-\overline{\nu}_e$.

We proceed to perform a similar fit but now with V_{us} as a free parameter, along with a' and b'. This fit is equivalent to the one recently performed in Ref. [6], except that in this reference the decay rates and g_1/f_1 ratios were used instead. The results of our fit correspond to the third column of Table II, labeled as Fit 1(b). There is a slight modification in the value of a' compared with the previous fit whereas b' remains practically unchanged. The fit yields $V_{us} = 0.2238 \pm 0.0019$, which is lower than the one of Ref. [6]. This time, $\chi^2 = 34.38$ for 14 degrees of freedom. Though χ^2 is reduced by around 4, this is not much and the fit is again far from being satisfactory. The lowering of χ^2 comes mainly from R in $\Sigma^- \to ne^-\overline{\nu}_e$ and $\Xi^- \to \Lambda e^-\overline{\nu}_e$, whose contributions are reduced by almost two and three, respectively. Still, α_ν in $\Lambda \to pe^-\overline{\nu}_e$ and R in $\Xi^- \to \Lambda e^-\overline{\nu}_e$ show worrisome deviations from the theoretical predictions.

We close this section by pointing out that the high χ^2 of these two fits is a clear evidence of $SU(3)$ symmetry breaking. We now proceed to analyze such effects by incorporating first- and second-order symmetry breaking into the axial-vector and vector form factors g_1 and f_1, respectively.

B. Symmetry breaking in g_1

To appreciate the effects of the departure from the exact $SU(3)$ flavor symmetry, we incorporate first-order symmetry breaking in g_1 through the parameters a', b' and c_{3-4}, while still keeping f_1,

f_2, and g_2 at their $SU(3)$ symmetric values. Fitting these parameters leads to the results displayed as Fit 2(a) of Table II, with $\chi^2 = 24.41$ for 13 degrees of freedom. The highest contributions to χ^2 come now from α_e ($\Delta\chi^2 = 2.70$) and α_ν ($\Delta\chi^2 = 6.80$) in $\Lambda \to pe^-\overline{\nu}_e$ and R ($\Delta\chi^2 = 2.55$), α_ν ($\Delta\chi^2 = 3.94$), and α_B ($\Delta\chi^2 = 2.54$) in $\Sigma^- \to ne^-\overline{\nu}_e$. Except for the remarkable improvement in the predictions of the observables in $\Xi^- \to \Lambda e^-\overline{\nu}_e$, whose combined contribution to χ^2 in this case amounts to less that 1.5, we observe only slight reductions in the contributions to χ^2 of the remaining observables, compared with Fit 1(a). As for the fitted parameters, again the leading parameter a' is order unity and b' is order $1/N_c$. The small effects due to symmetry breaking can be seen mainly in the new value of b' compared to the $SU(3)$ symmetric fit, and in the parameters c_{3-4}, which are small or even smaller than expected from first-order symmetry breaking (our rough measure of symmetry breaking is $\epsilon \sim 30\%$) and factors of $1/N_c$. The values of the best fit parameters are consistent with previous works [9, 16].

In a similar fashion, we can attempt to extract the value of V_{us} in this context. The results are displayed in the column labeled as Fit 2(b) of Table II. The fitted parameters change a little and $V_{us} = 0.2230 \pm 0.0019$ with $\chi^2 = 21.79$ for 12 degrees of freedom. The contributions to χ^2 come from the very same observables as in the previous fit, with some minor changes in the observables other than the usual ones which systematically have the highest contributions to χ^2. Regardless of the still high χ^2, we can observe that incorporating first-order symmetry breaking corrections into g_1 lowers the predicted value of V_{us} compared to the case with no symmetry breaking at all, Fit 1(b). This fact indeed is crucial to reinforce our initial argument that exact $SU(3)$ no longer provides an acceptable fit.

Let us now find out how the inclusion of second-order symmetry breaking into f_1 impacts on the various observables, before drawing any conclusions.

C. Symmetry breaking in both f_1 and g_1

In this section we incorporate second-order symmetry breaking into the vector form factor f_1, so that it is no longer fixed at its $SU(3)$ symmetric value $f_1^{SU(3)}$. We expect these effects to be second-order in symmetry breaking (roughly $\epsilon^2 \sim 9\%$) according to the Ademollo-Gatto theorem. For this fit, then, the parameters v_{1-3} of f_1 enter into play, simultaneously with a', b', and c_{3-4} of g_1 while f_2 and g_2 remain fixed by exact $SU(3)$ symmetry. The best fit parameters are displayed as Fit 3(a) of Table II, with $\chi^2 = 17.85$ for 10 degrees of freedom. When V_{us} is also allowed to

TABLE II: Best fitted parameters for the vector and axial-vector form factors. The rates and asymmetry coefficients were used.

	Fit 1		Fit 2		Fit 3	
	(a)	(b)	(a)	(b)	(a)	(b)
V_{us}	Fixed	0.2238 ± 0.0019	Fixed	0.2230 ± 0.0019	Fixed	0.2199 ± 0.0026
v_1					0.00 ± 0.03	0.00 ± 0.04
v_2					0.02 ± 0.03	0.02 ± 0.03
v_3					-0.01 ± 0.01	-0.01 ± 0.01
a'	0.80 ± 0.01	0.78 ± 0.01	0.71 ± 0.03	0.70 ± 0.03	0.72 ± 0.03	0.72 ± 0.03
b'	-0.07 ± 0.01	-0.07 ± 0.01	-0.08 ± 0.01	-0.08 ± 0.01	-0.08 ± 0.01	-0.08 ± 0.01
c_3			0.03 ± 0.02	0.03 ± 0.02	0.03 ± 0.02	0.03 ± 0.02
c_4			0.06 ± 0.02	0.06 ± 0.02	0.05 ± 0.02	0.05 ± 0.02
χ^2/dof	38.63/15	34.38/14	24.41/13	21.79/12	17.85/10	17.85/9

be a free parameter, we obtain the results displayed in the last column of that table, labeled as Fit 3(b). The fit yields $V_{us} = 0.2199 \pm 0.0026$. In both cases, the best fit parameters are as expected from the $1/N_c$ expansion predictions. As a matter of fact, hereafter we will loosely refer to Fit 3(b) as the final fit.

In Table III we display the predicted form factors corresponding to the final fit. These form factors yield the predicted observables shown in Table IV. Going through the latter table and comparing its entries with the predictions produced by Fit 1(a), namely the $SU(3)$ fit, we can find some improvements all over except in a well identified subset of data which carries most of the weight of the deviations from the theoretical expectations. This subset is formed by the angular asymmetries α_e, α_ν, and α_B of both processes $\Lambda \to pe^-\overline{\nu}_e$ and $\Sigma^- \to ne^-\overline{\nu}_e$, which remain still too far from the current experimental data. Particularly, there has been no noticeably change, in any fit performed, in either α_ν or α_e of the former decay, despite the important reduction of χ^2 by more than half from the initial fit to the final one.

V. FITS TO EXPERIMENTAL DATA: DECAY RATES AND g_1/f_1 RATIOS

We can now attempt to make a comparison between theory and experiment in another way. This time we can perform a global fit by using the decay rates and measured g_1/f_1 ratios, the latter also contained in Table I. We proceed as before, namely, we first perform an $SU(3)$ fit, next we include first- and second-order symmetry breaking effects in the axial-vector and vector form

TABLE III: Predicted form factors. The quoted errors come from the fit only.

	Fit 1(a)			Fit 1(b)	Fit 2(a)	Fit 2(b)	Fit 3(b)	
Transition	f_1	f_2	g_1	g_1	g_1	g_1	f_1	g_1
$\Lambda \to p$	-1.22	-1.10	-0.89 ± 0.01	-0.87 ± 0.01	-0.88 ± 0.01	-0.87 ± 0.01	-1.25 ± 0.02	-0.88 ± 0.02
$\Sigma^- \to n$	-1.00	1.02	0.34 ± 0.01	0.33 ± 0.01	0.35 ± 0.01	0.34 ± 0.01	-1.04 ± 0.02	0.34 ± 0.01
$\Xi^- \to \Lambda$	1.22	-0.07	0.24 ± 0.01	0.23 ± 0.01	0.40 ± 0.04	0.38 ± 0.04	1.28 ± 0.06	0.37 ± 0.05
$\Xi^- \to \Sigma^0$	0.71	1.31	0.89 ± 0.01	0.87 ± 0.01	0.92 ± 0.05	0.91 ± 0.05	0.75 ± 0.04	0.93 ± 0.06
$\Xi^0 \to \Sigma^+$	1.00	1.85	1.26 ± 0.01	1.23 ± 0.02	1.30 ± 0.08	1.26 ± 0.08	1.07 ± 0.05	1.31 ± 0.08

TABLE IV: Theoretical predictions for five $|\Delta S| = 1$ hyperon semileptonic decays and their contributions to the total χ^2. The rates and angular coefficients were mainly used in the fit. The units of R are 10^6 s^{-1}.

	$\Lambda \to pe^-\overline{\nu}_e$		$\Sigma^- \to ne^-\overline{\nu}_e$		$\Xi^- \to \Lambda e^-\overline{\nu}_e$		$\Xi^- \to \Sigma^0 e^-\overline{\nu}_e$		$\Xi^0 \to \Sigma^+ e^-\overline{\nu}_e$	
	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$
R	3.16	0.0	6.87	0.0	3.40	0.0	0.54	0.1	0.98	0.1
$\alpha_{e\nu}$	-0.01	0.1	0.36	0.1	0.51	0.0				
α_e	0.03	2.3	-0.62	0.9						
α_ν	0.97	6.5	-0.35	3.8						
α_B	-0.59	1.7	0.65	2.0						
A					0.63	0.1				
g_1/f_1	0.71		-0.33		0.29		1.23	0.1	1.23	0.2

factors, respectively, along the lines of Secs. IV A-IV C. The results are all displayed in Table V as Fits 4, 5, and 6. Hereafter, let us refer to Fit 6(b) –the one with symmetry breaking effects in f_1 and g_1 and V_{us} as a free parameter– as the alternative fit.

The parameters involved in the fits follow a similar behavior as the preceding ones, so there is no need to reproduce here the predicted form factors. Instead, we proceed to display in Table V the predicted observables obtained within the alternative fit. Looking through Tables V and VI, we find a very good agreement between the final fit and the alternative one. We also observe that the value of V_{us} is systematically reduced from the $SU(3)$ prediction by including symmetry breaking effects in the form factors. The alternative fit yields $V_{us} = 0.2200 \pm 0.0026$, in good agreement with the final fit value. Indeed, taking into account the low χ^2 of the alternative fit, we might conclude that it is satisfactory. This conclusion is misleading because fitting the rates and g_1/f_1 ratios hides the deviations in the polarization data found in Sec. IV. This interesting finding cannot be elucidated otherwise.

TABLE V: Best fitted parameters for the vector and axial-vector form factors. The rates and g_1/f_1 ratios were used.

	Fit 4		Fit 5		Fit 6	
	(a)	(b)	(a)	(b)	(a)	(b)
V_{us}	Fixed	0.2230 ± 0.0019	Fixed	0.2222 ± 0.0019	Fixed	0.2200 ± 0.0026
v_1					-0.02 ± 0.04	-0.02 ± 0.04
v_2					0.03 ± 0.03	0.03 ± 0.03
v_3					-0.01 ± 0.01	-0.01 ± 0.01
a'	0.81 ± 0.01	0.80 ± 0.01	0.73 ± 0.03	0.73 ± 0.03	0.75 ± 0.03	0.75 ± 0.03
b'	-0.08 ± 0.01	-0.08 ± 0.01	-0.09 ± 0.01	-0.08 ± 0.01	-0.08 ± 0.01	-0.08 ± 0.01
c_3			0.02 ± 0.02	0.02 ± 0.02	0.02 ± 0.02	0.02 ± 0.02
c_4			0.06 ± 0.02	0.05 ± 0.02	0.04 ± 0.02	0.04 ± 0.02
χ^2/dof	16.50/8	13.91/7	5.27/6	3.86/5	0.72/3	0.72/2

TABLE VI: Theoretical predictions for five $|\Delta S| = 1$ hyperon semileptonic decays and their contributions to the total χ^2. The rates and g_1/f_1 ratios were used in the fit. The units of R are 10^6 s^{-1}.

	$\Lambda \to pe^-\overline{\nu}_e$		$\Sigma^- \to ne^-\overline{\nu}_e$		$\Xi^- \to \Lambda e^-\overline{\nu}_e$		$\Xi^- \to \Sigma^0 e^-\overline{\nu}_e$		$\Xi^0 \to \Sigma^+ e^-\overline{\nu}_e$	
	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$
R	3.16	0.0	6.85	0.0	3.40	0.0	0.55	0.0	0.99	0.2
$\alpha_{e\nu}$	-0.02		0.34		0.55					
α_e	0.02		-0.63							
α_ν	0.98		-0.35							
α_B	-0.59		0.66							
A					0.66					
g_1/f_1	0.72	0.0	-0.34	0.0	0.26	0.1	1.22	0.2	1.22	0.2

VI. COMPARING WITH MODELS OF $SU(3)$ SYMMETRY BREAKING

Various treatments of $SU(3)$ breaking effects in the HSD couplings have been explicitly computed in order to understand the deviations from exact $SU(3)$. We of course do not pretend to be exhaustive, but a representative selection of such treatments can be found in Refs. [20, 21, 22, 23]. It is hard to assess the success of these models, mainly because their approaches and/or assumptions are rather different. Some rely on quark models and others on chiral perturbation theory or some variations of such methods. They explicitly provide $SU(3)$ breaking corrections to f_1, which are summarized in Table VII as the ratios $f_1/f_1^{SU(3)}$. We also include in this table the patterns obtained in the present paper with the final fit of Sec. IV C and

TABLE VII: Symmetry breaking pattern for f_1. The entries correspond to $f_1/f_1^{\mathrm{SU(3)}}$.

Transition	Fit 3(b)	Fit 6(b)	R.F.M *et. al* [9]	Anderson and Luty [20]	Donoghue *et. al.* [21]	Krause [22]	Schlumpf [23]
$\Lambda \to p$	1.02±0.02	1.02±0.02	1.02±0.02	1.024	0.987	0.943	0.976
$\Sigma^- \to n$	1.04±0.02	1.04±0.03	1.04±0.02	1.100	0.987	0.987	0.975
$\Xi^- \to \Lambda$	1.04±0.04	1.04±0.04	1.10±0.04	1.059	0.987	0.957	0.976
$\Xi^- \to \Sigma^0$	1.07±0.05	1.08±0.05	1.12±0.05	1.011	0.987	0.943	0.976
$\Xi^0 \to \Sigma^+$	1.07±0.05	1.08±0.05	1.12±0.05				
V_{us}	0.2199±0.0026	0.2200±0.0026	0.2194±0.0023	0.2177±0.0019	0.2244±0.0019	0.2274±0.0019	0.2256±0.0019

the alternative fit of Sec. V, together with the one of Ref. [9], which was obtained under the same assumptions of this work but by performing a combined fit of HSD (both $\Delta S = 0$ and $|\Delta S| = 1$ data) and pionic decays of the decuplet baryons. With this information, we can proceed to find out the trends of these models toward the determination of V_{us}. As for g_1, Refs. [20, 21] also provide its breaking pattern. In order to make a comparison on an equal footing of the four models we find more convenient to leave g_1 as a free parameter. We now must resort to a model-independent determination of g_1 which allows the extraction of symmetry breaking corrections from experiment in a way as general as possible. For this purpose we can use the $1/N_c$ expansion and fit the parameters a', b', and c_{3-4}, or we can adapt the approach of Ref. [10], which assumes that symmetry breaking comes from the eight component of an octet in the strong-interaction Hamiltonian. In this scheme, g_1 can be parametrized in terms of seven quantities, namely, $\tilde{F}$, $\tilde{D}$, A_1, B_1, C_1, D_1 and E_1, the first two quantities corresponding to the exact symmetric limit [25]. For the fits we include the decay rates, the angular coefficients and the ratio g_1/f_1 of $\Xi^- \to \Sigma^0 e^- \overline{\nu}$, leaving out the experimental information on the decay $\Xi^0 \to \Sigma^+ e^- \overline{\nu}$ because the ratio $f_1/f_1^{SU(3)}$ is not provided by the models.

The values of V_{us} extracted within these models are listed in the bottom row of Table VII. The fits in general are stable but produce χ^2/dof higher than two. Among the models, only Refs. [21, 23] quote values of V_{us} and our predictions agree well with theirs. Starting with the V_{us} obtained in the frame of exact $SU(3)$ symmetry in Fit 1(b), namely, $V_{us} = 0.2238 \pm 0.0019$, we can observe immediately by looking through Table VII that a pattern of symmetry breaking such as $f_1/f_1^{SU(3)} < 1$ will systematically increase V_{us} from the former value, whereas the opposite trend occurs when $f_1/f_1^{SU(3)} > 1$. Let us now discuss the consequences of these findings in our concluding section.

VII. CONCLUSIONS

So far we can establish two interesting findings from our analysis. The first one concerns the issue that the assumption of exact $SU(3)$ symmetry, as customarily used to compare theory and experiment in HSD, is questionable due to the poor fits it produces. The second one is related to the fact that deviations from the exact $SU(3)$ limit, accounted for in the form factors f_1 and g_1 are indeed important in order to reliably determine V_{us} from HSD, which can rival in precision with the one from K_{e3} decays. The value currently admitted of V_{us} for the latter decays is given in Eq. (1). In our analysis, we have performed a series of fits under several assumptions in the context of exact and broken $SU(3)$ symmetry. If we consider the limit of exact symmetry, the fit produces $V_{us} = 0.2238 \pm 0.0019$, which is higher than (1). By including first-order symmetry breaking effects into the axial-vector form factors g_1, the fit now yields $V_{us} = 0.2230 \pm 0.0019$, which is still higher. However, the main conclusion we can draw from the above analysis is that a reconciliation between these two determinations can be obtained only through second-order breaking effects of a few percent in the leading vector form factors f_1, which always increase their magnitudes over their exact $SU(3)$ symmetry predictions $f_1^{SU(3)}$, as displayed in Table VII, second column, and also found in previous works [9, 19, 20]. Therefore, experimental data seems to favor this trend. From this, the value of V_{us} we can extract from hyperon semileptonic decays is

$$V_{us} = 0.2199 \pm 0.0026, \tag{20}$$

which is comparable to (1) and indeed, agrees very well with the value of $V_{us} = 0.2208 \pm 0.0034$ obtained very recently from hadronic τ decays [24].

We consider pertinent to remark that the experimental information used in the fit was constituted mainly by the decay rates and angular correlation and spin-asymmetry coefficients. Although we have performed similar fits by using the rates and the g_1/f_1 ratios, we should point out that the total χ^2 corresponding to them is small when symmetry breaking effects into the leading form factors are included. If these were the only pieces of data available to us, instead of the angular coefficients, we would be prompted to conclude that the exact $SU(3)$ symmetry limit is in very good agreement with experiment. It is clear that using the angular coefficients, instead of the g_1/f_1 ratios, not only avoids inconsistencies but provides a more sensitive test. From this point of view we can conclude that, although HSD data are rather scarce, they are restrictive enough to make us look into the exact symmetry limit assumption with more care.

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APPENDIX A: NUMERICAL FORMULAS FOR HSD INTEGRATED OBSERVABLES

In this Appendix we provide updated numerical formulas for the uncorrected transition rates and angular coefficients of the five $|\Delta S| = 1$ HSD dealt with in the present paper, namely, R^0, and $\alpha_{e\nu}^0$, α_ν^0, α_e^0, α_B^0, and A^0. The theoretical expressions used for these integrated observables are those computed in Ref. [10], where one can find further details about the kinematical region of integration. The inputs for the numerical evaluation are the value of the weak coupling constant G and the experimental values of the hyperon masses, which are all found in Ref. [2]. The slope parameters of the leading form factors are defined in Eq. (12). Neither radiative corrections nor V_{us} are included at all.

The total decay rate R^0, being quadratic in the form factors, can be written in the most general form as

$$R^0 = \sum_{i\leq j=1}^{6} a_{ij}^R f_i f_j + \sum_{i\leq j=1}^{6} b_{ij}^R (f_i \lambda_{f_j} + f_j \lambda_{f_i}), \tag{A1}$$

where dipole parametrizations similar to Eq. (12) have been assumed for all form factors, which introduce in total six slope parameters λ_{f_i}. For the sake of shortening Eq. (A1), we have momentarily redefined $g_1 = f_4$, $g_2 = f_5$, $g_3 = f_6$, $\lambda_{g_1} = \lambda_{f_4}$, $\lambda_{g_2} = \lambda_{f_5}$, and $\lambda_{g_3} = \lambda_{f_6}$. Notice that the restriction $i \leq j$ reduces each sum in Eq. (A1) to 21 terms. Similar expressions to Eq. (A1) also hold for the products $R^0\alpha^0$, where α^0 is any of the angular coefficients defined above. Once R^0 and $R^0\alpha^0$ are determined, α^0 is obtained straightforwardly.

The integrated observables have been organized in Tables VIII-XII. Although we have completely computed all 42 terms involved in Eq. (A1), we have not listed neither the contributions of f_3 and g_3 nor the ones from the slope parameters λ_{f_2}, λ_{g_2}, λ_{f_3}, and λ_{g_3}. The entries have been truncated to four decimal places so that we have also omitted contributions lower than 10^{-5}, which we consider small compared to the leading ones.

For a particular decay, the coefficients a_{ij}^R and b_{ij}^R of R^0 in Eq. (A1) can be easily read off from

TABLE VIII: Numerical formulas for some integrated observables of $\Lambda \to pe^-\overline{\nu}$ decay. The units of R^0 are 10^6 s^{-1}.

	R^0	$R^0\alpha^0_{e\nu}$	$R^0\alpha^0_\nu$	$R^0\alpha^0_e$	$R^0\alpha^0_B$	R^0A^0
f_1f_1	15.2774	12.5169	0.9291	−0.9290		
f_2f_2	0.2200	−0.1553	0.0860	−0.0860		
g_1g_1	45.4432	−22.4798	31.0949	−31.0937		−0.0008
g_2g_2	0.6558	−0.8591	0.5217	−0.5217		
f_1f_2	0.3580	−0.1982	1.7571	−1.7570		0.0001
g_1g_2	−9.6247	11.1472	−8.2254	8.2252		0.0002
f_1g_1		−0.0002	28.6966	28.6951	−35.7929	41.9279
f_1g_2			−1.3993	−1.3991	1.6632	−2.8165
f_2g_1		−0.0004	−1.3992	−1.3995	3.8473	2.8167
f_2g_2		0.0001	0.2681	0.2681	−0.6256	−0.4911
$f_1\lambda_{f_1}$	0.2211	−0.0570	0.0201	−0.0201		
$g_1\lambda_{g_1}$	1.0926	−1.4647	0.8915	−0.8915		
$f_1\lambda_{g_1}+g_1\lambda_{f_1}$			0.2011	0.2011	−0.2921	0.3683

TABLE IX: Numerical formulas for some integrated observables of $\Sigma^- \to ne^-\overline{\nu}$ decay. The units of R^0 are 10^6 s^{-1}.

	R^0	$R^0\alpha^0_{e\nu}$	$R^0\alpha^0_\nu$	$R^0\alpha^0_e$	$R^0\alpha^0_B$	R^0A^0
f_1f_1	90.5903	67.3788	7.8358	−7.8356		0.0002
f_2f_2	2.3860	−1.8647	0.9864	−0.9864		
g_1g_1	267.2903	−147.6648	184.5359	−184.5326		−0.0024
g_2g_2	7.0659	−9.6050	5.6662	−5.6662		
f_1f_2	3.9980	−2.5400	14.5368	−14.5364		0.0003
g_1g_2	−76.5914	92.9306	−66.0522	66.0512		0.0007
f_1g_1		−0.0008	165.5089	165.5046	−206.0087	250.6750
f_1g_2		0.0002	−10.5392	−10.5383	12.1688	−23.2535
f_2g_1		−0.0014	−10.5388	−10.5398	30.1444	23.2542
f_2g_2		0.0003	2.7993	2.7994	−6.7007	−5.4083
$f_1\lambda_{f_1}$	2.4092	−0.8622	0.3096	−0.3096		
$g_1\lambda_{g_1}$	11.7689	−16.3427	9.6693	−9.6693		−0.0001
$f_1\lambda_{g_1}+g_1\lambda_{f_1}$			2.0996	2.0995	−3.0382	4.0562

the second column in each table. Similarly, in the third column one can read off the coefficients $a_{ij}^{R\alpha_{e\nu}}$ and $b_{ij}^{R\alpha_{e\nu}}$ of the product $R^0\alpha^0_{e\nu}$, and so on. These numerical formulas are the ones used in the fits to experimental data performed in the present paper.

TABLE X: Numerical formulas for some integrated observables of $\Xi^- \to \Lambda e^- \overline{\nu}$ decay. The units of R^0 are 10^6 s^{-1}.

	R^0	$R^0\alpha^0_{e\nu}$	$R^0\alpha^0_{\nu}$	$R^0\alpha^0_{e}$	$R^0\alpha^0_{B}$	R^0A^0
f_1f_1	32.1282	26.4627	1.9066	-1.9065		0.0001
f_2f_2	0.4433	-0.3108	0.1726	-0.1726		
g_1g_1	95.6040	-46.9613	65.3824	-65.3805		-0.0013
g_2g_2	1.3215	-1.7273	1.0508	-1.0508		
f_1f_2	0.7199	-0.3951	3.6100	-3.6099		0.0001
g_1g_2	-19.8176	22.8864	-16.9273	16.9268		0.0003
f_1g_1		-0.0003	60.4433	60.4410	-75.3978	88.1274
f_1g_2			-2.8903	-2.8900	3.4407	-5.7868
f_2g_1		-0.0006	-2.8901	-2.8906	7.9291	5.7871
f_2g_2		0.0001	0.5413	0.5413	-1.2611	-0.9883
$f_1\lambda_{f_1}$	0.4454	-0.1121	0.0394	-0.0394		
$g_1\lambda_{g_1}$	2.2017	-2.9451	1.7958	-1.7957		
$f_1\lambda_{g_1}+g_1\lambda_{f_1}$			0.4060	0.4060	-0.5899	0.7413

TABLE XI: Numerical formulas for some integrated observables of $\Xi^- \to \Sigma^0 e^- \overline{\nu}$ decay. The units of R^0 are 10^6 s^{-1}.

	R^0	$R^0\alpha^0_{e\nu}$	$R^0\alpha^0_{\nu}$	$R^0\alpha^0_{e}$	$R^0\alpha^0_{B}$	R^0A^0
f_1f_1	3.3767	3.0211	0.1192	-0.1192		
f_2f_2	0.0183	-0.0114	0.0067	-0.0067		
g_1g_1	10.0998	-4.3598	6.8423	-6.8418		-0.0004
g_2g_2	0.0547	-0.0687	0.0431	-0.0431		
f_1f_2	0.0288	-0.0134	0.2303	-0.2303		
g_1g_2	-1.3109	1.4382	-1.1093	1.1093		
f_1g_1		-0.0001	6.5150	6.5144	-8.1375	9.1718
f_1g_2			-0.2016	-0.2015	0.2454	-0.3694
f_2g_1		-0.0001	-0.2015	-0.2016	0.5327	0.3694
f_2g_2			0.0231	0.0231	-0.0526	-0.0401
$f_1\lambda_{f_1}$	0.0183	-0.0028	0.0010	-0.0010		
$g_1\lambda_{g_1}$	0.0912	-0.1173	0.0738	-0.0738		
$f_1\lambda_{g_1}+g_1\lambda_{f_1}$			0.0173	0.0173	-0.0253	0.0301

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TABLE XII: Numerical formulas for some integrated observables of $\Xi^0 \to \Sigma^+ e^- \overline{\nu}$ decay. The units of R^0 are 10^6 s^{-1}.

	R^0	$R^0\alpha^0_{e\nu}$	$R^0\alpha^0_{\nu}$	$R^0\alpha^0_{e}$	$R^0\alpha^0_{B}$	$R^0 A^0$
$f_1 f_1$	2.9853	2.6777	0.1031	-0.1031		
$f_2 f_2$	0.0155	-0.0096	0.0057	-0.0057		
$g_1 g_1$	8.9302	-3.8371	6.0480	-6.0475		-0.0003
$g_2 g_2$	0.0464	-0.0582	0.0366	-0.0366		
$f_1 f_2$	0.0244	-0.0113	0.1993	-0.1993		
$g_1 g_2$	-1.1358	1.2438	-0.9608	0.9608		
$f_1 g_1$			5.7644	5.7639	-7.2001	8.1058
$f_1 g_2$			-0.1749	-0.1749	0.2131	-0.3197
$f_2 g_1$		-0.0001	-0.1749	-0.1750	0.4618	0.3197
$f_2 g_2$			0.0197	0.0197	-0.0446	-0.0340
$f_1 \lambda_{f_1}$	0.0155	-0.0023	0.0008	-0.0008		
$g_1 \lambda_{g_1}$	0.0774	-0.0995	0.0627	-0.0627		
$f_1 \lambda_{g_1} + g_1 \lambda_{f_1}$			0.0147	0.0147	-0.0215	0.0255

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[25] Actually, $\tilde{F}$ and $\tilde{D}$ are related to the conventional reduced form factors F and D as $F = \tilde{F}/\sqrt{6}$ and $D = -\sqrt{3/10}\tilde{F}$. We have put a tilde on them to avoid confusion.